# A Radon-transzformáció valós idejű webes vizualizációja

Lócsi Levente

ELTE IK Numerikus Analízis Tanszék

Bolyai Kollégium, Informatika Szeminárium
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EFOP-3.6.3-VEKOP-16-2017-00001

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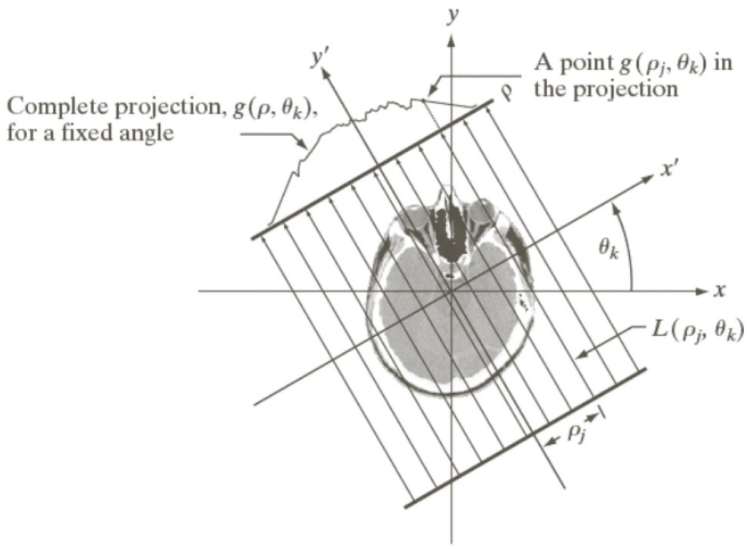
# Table of Contents

# Table of Contents

# The Radon transform

Johann Radon, 1917: *Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten*

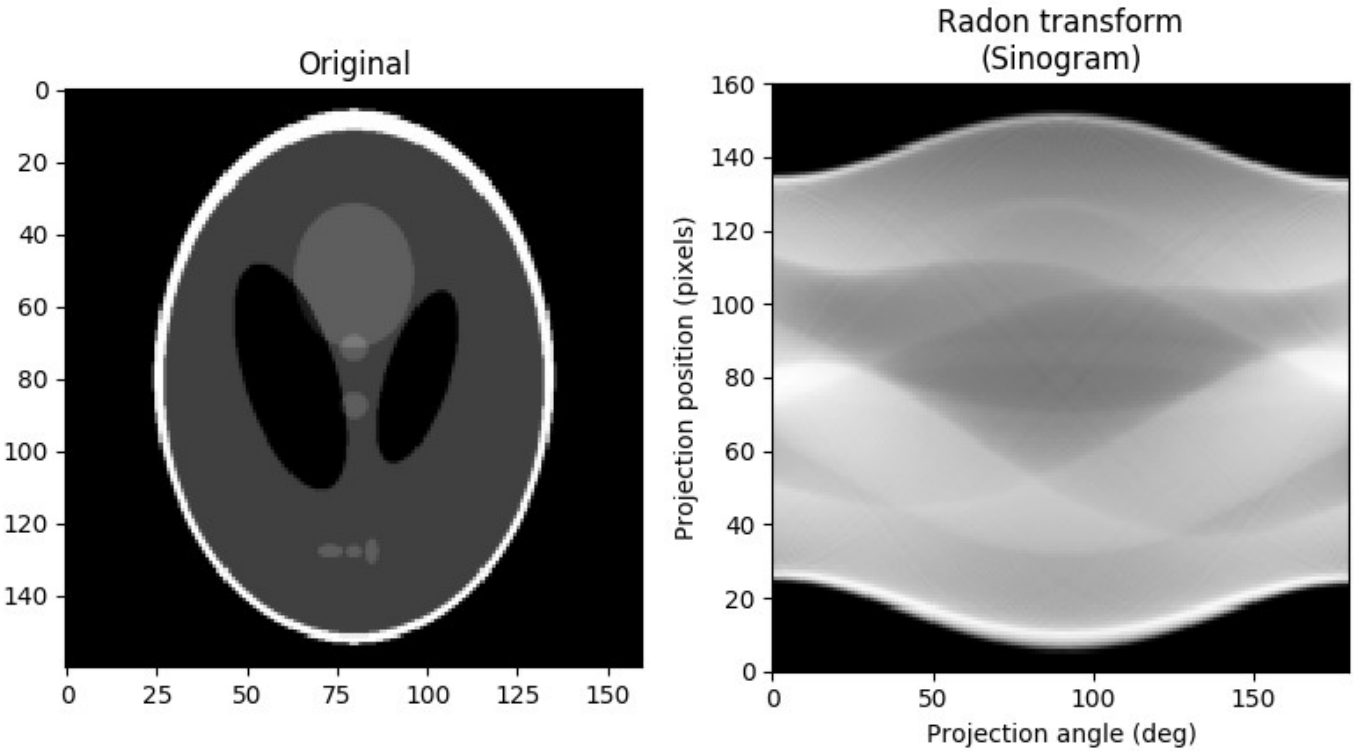
$$\mathcal{R}f(\varphi, r) = \int_{\mathbb{R}} f(r \cdot \cos\varphi - t \cdot \sin\varphi, r \cdot \sin\varphi + t \cdot \cos\varphi)\,\mathrm{d}t$$




Computer Tomography (CT) imaging; inverse Radon transform
Related to the Fourier transform

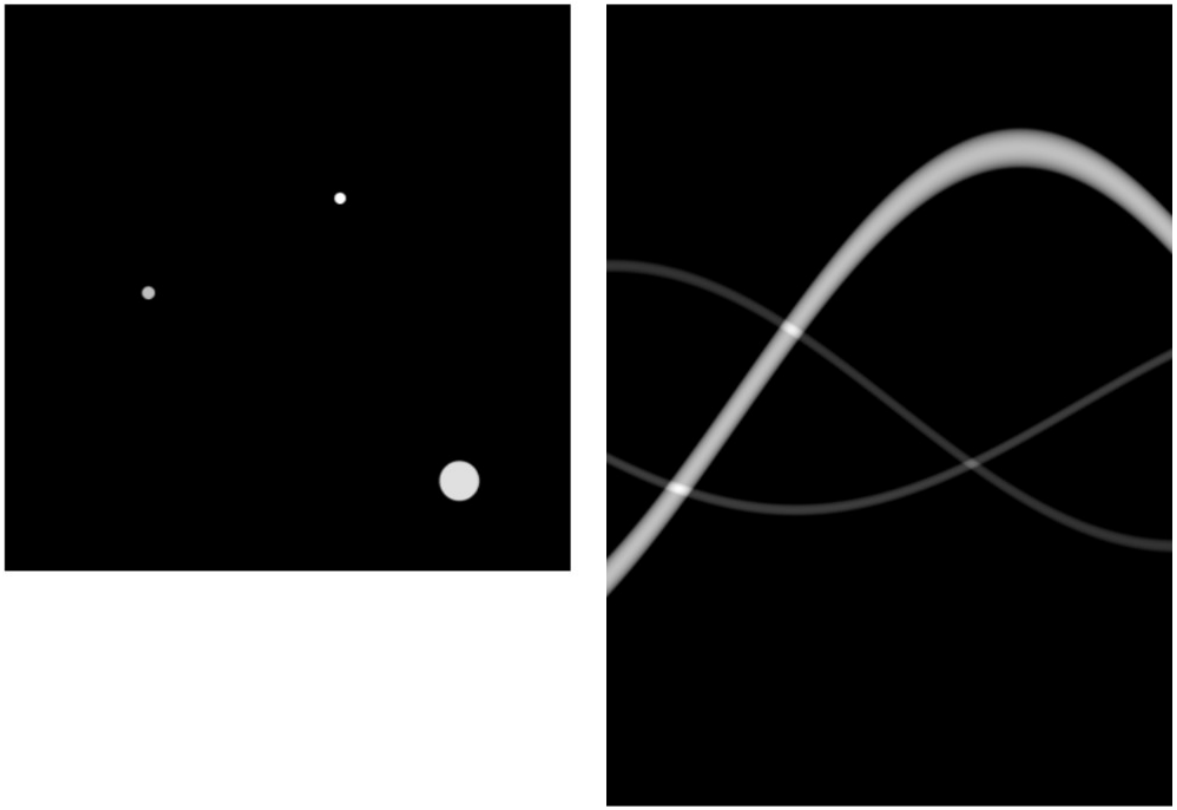
# Tha Radon transform – Example 1

The Shepp–Logan brain phantom (as in scikit-image / python)



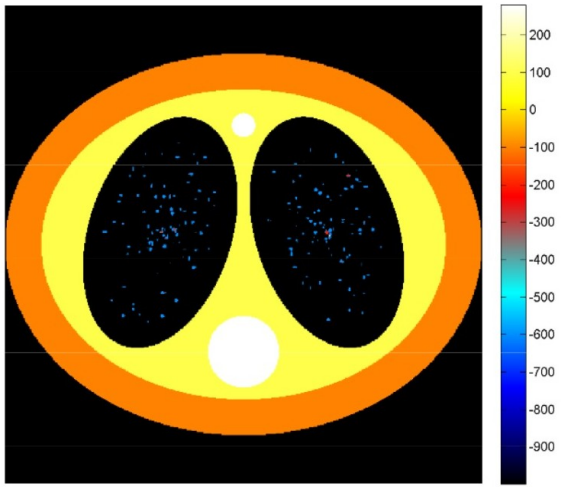
# Tha Radon transform – Example 2

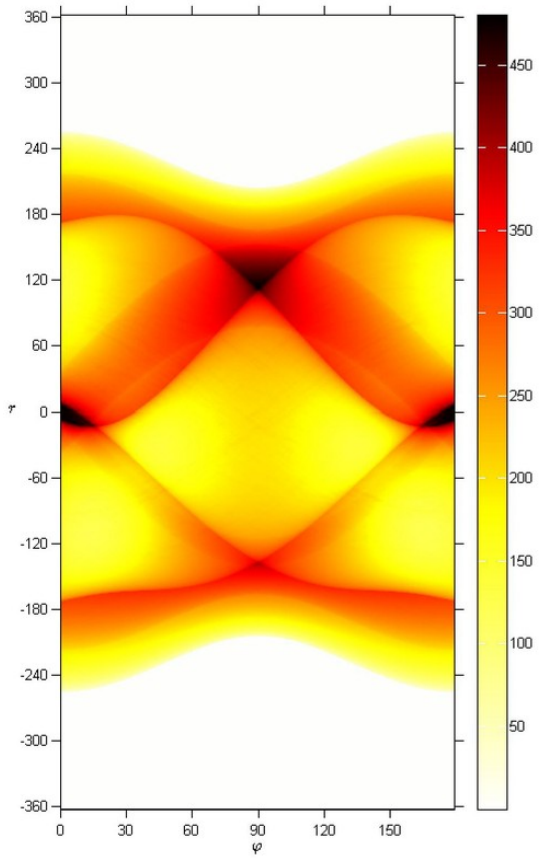
Three small circles (as in the server-side approach)



# Tha Radon transform – Example 3

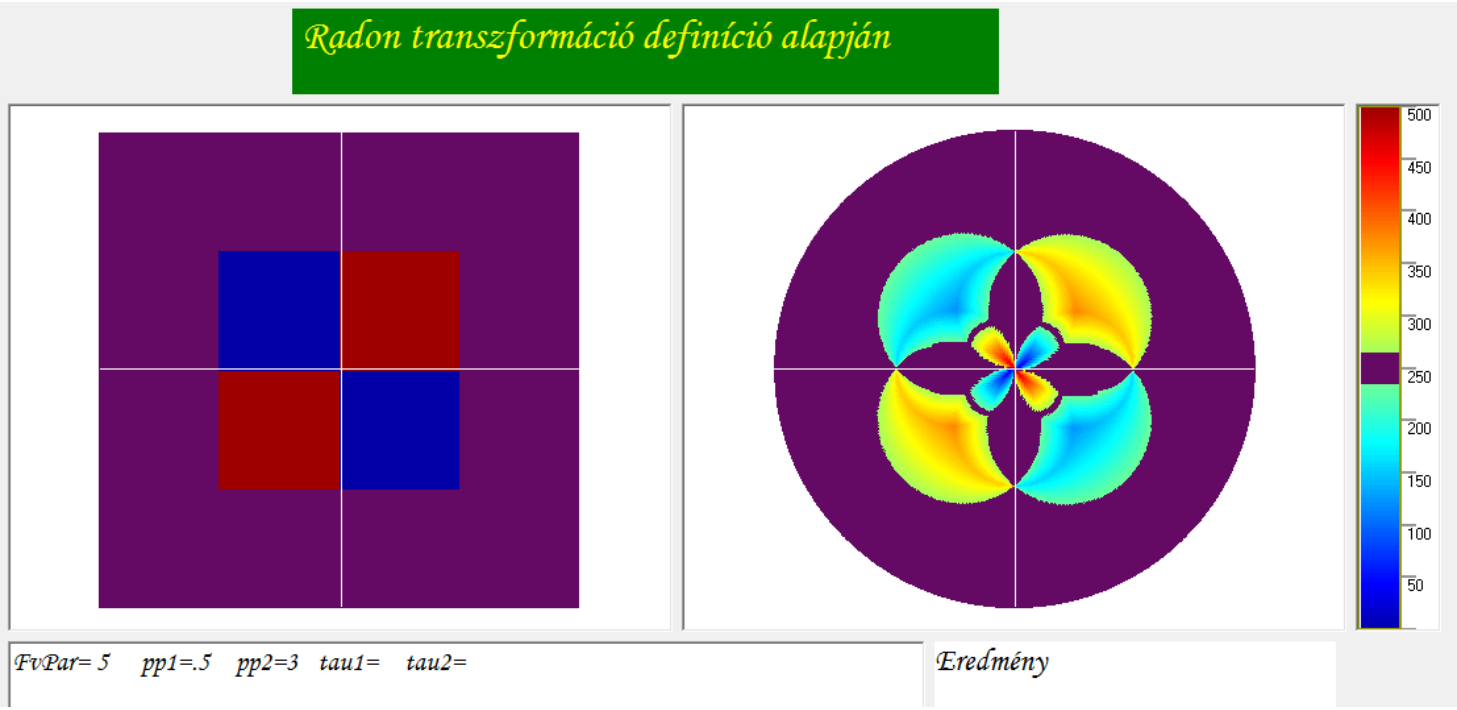
The Bognár lung phantom (2016, IJATES[2])



# Tha Radon transform – Example 4

Haar functions and grids by Prof. Ferenc Schipp (VB)
polar coordinates (instead of Cartesian)




See also `http://numanal.inf.elte.hu/~schipp/Jegyzetek/RadonTr.pdf`

# Motivation

Visualization is important

- Education, Science communication
- Understand basic properties of the transform
- Aesthetics

**Our goal:** A visualization method for the Radon transform

- Real-time, interactive scene editing and transform
- Easily accessible to everyone

Such visualization existed in the case of the Fourier transform (e.g. "Fourifier")

`https://ejectamenta.com/imaging-experiments/fourifier/`

# Table of Contents

# Matlab, Octave and co.

Built-in tools

- Matlab: Image Processing Toolbox
- Octave: image package (pkg load image)

```
image = zeros(400,400);
image(150:250,150:250) = 1;
tr = radon(image);
imagesc(tr)
```

octave-online, Mathematica,
WolframAlpha (try: radon(square))

# Matlab, Octave and co. – Verdict

| | Matlab & co. |
|---|---|
| detailed transform | ✓ |
| easily accessible | ✗ |
| interactive editing | ✗ |
| real-time transform | ✗ |

# Our first approach

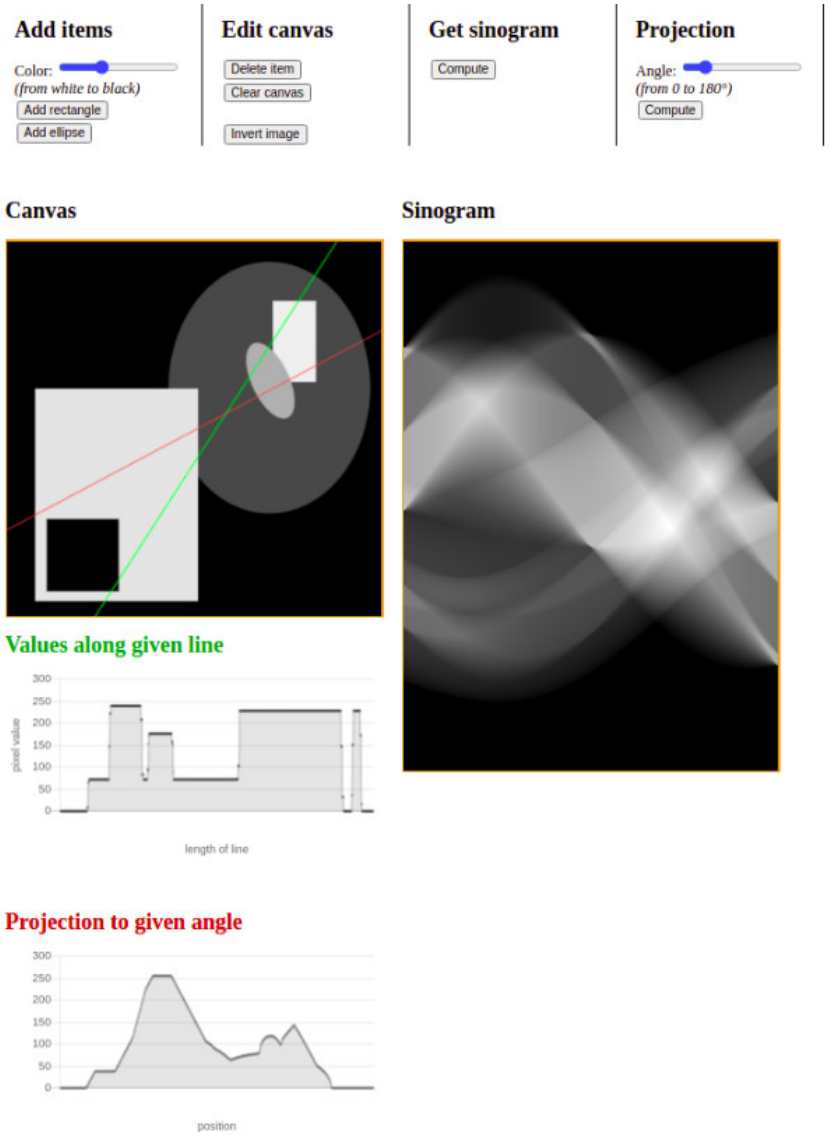
**A web-based, server-side, numerical approach**

Zsófia Gál: *Theory, visualization and application of the Radon transform* (MSc thesis, 2020, in Hungarian)

- client: html+css+javascript, interactive editing, fabric.js
- server: python, numpy, scikit-image, numerical calculation
- values along given line, projection to given angle

- security issues, not available online

# Our first approach – Demo



# Our first approach – Verdict

| | Server, Num |
|---|---|
| detailed transform | ✓ |
| easily accessible | ✗ |
| interactive editing | ✓ |
| real-time transform | ✗ |

# Our second approach

**A web-based, client-side, analytical approach**

Levente Lócsi: *Real-time web-based visualization of the Radon transform* (presentation, MaCS 2020, in English)

- client: html+css+javascript, interactive editing, fabric.js
- analytic calculation of positions of points of interest
- tradeoff: no detailed transform, "just" its support

# Our second approach – Some math

For an ellipse centered at $(0,0)$ with principal axes $a$ and $b$:

$$\mathcal{R}f(\varphi, r) \neq 0 \quad \Longleftrightarrow \quad r_{\text{min}}(\varphi) \leq r \leq r_{\text{max}}(\varphi) \quad (\varphi \in \mathbb{R})$$

$$r_{\text{min,max}}(\varphi) = \pm\sqrt{b^2 \cdot \sin^2\varphi + a^2 \cdot \cos^2\varphi}$$

If it is centered at $(x_0, y_0)$ and rotated with angle $\alpha$:

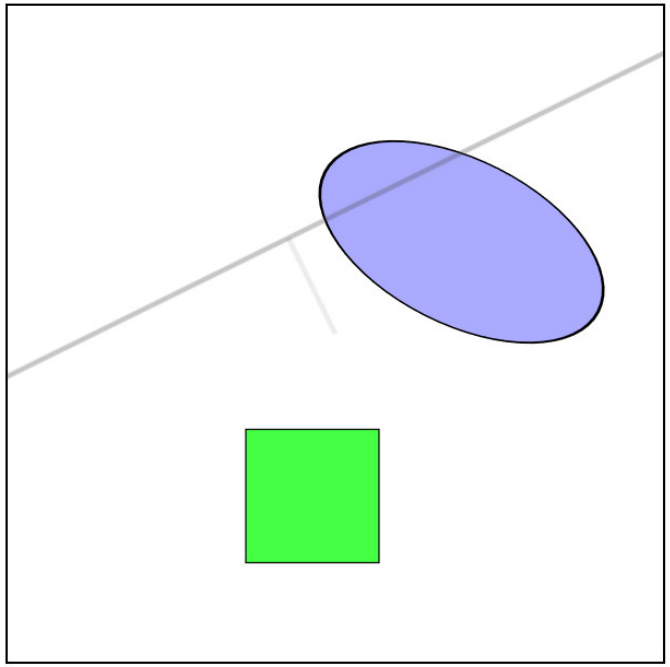
$$r_{\text{min,max}}(\varphi) = \sqrt{x_0^2 + y_0^2} \cdot \cos\left(\varphi - \operatorname{atan}\frac{y}{x}\right) \pm$$

$$\pm\sqrt{b^2 \cdot \sin^2(\varphi - \alpha) + a^2 \cdot \cos^2(\varphi - \alpha)}$$

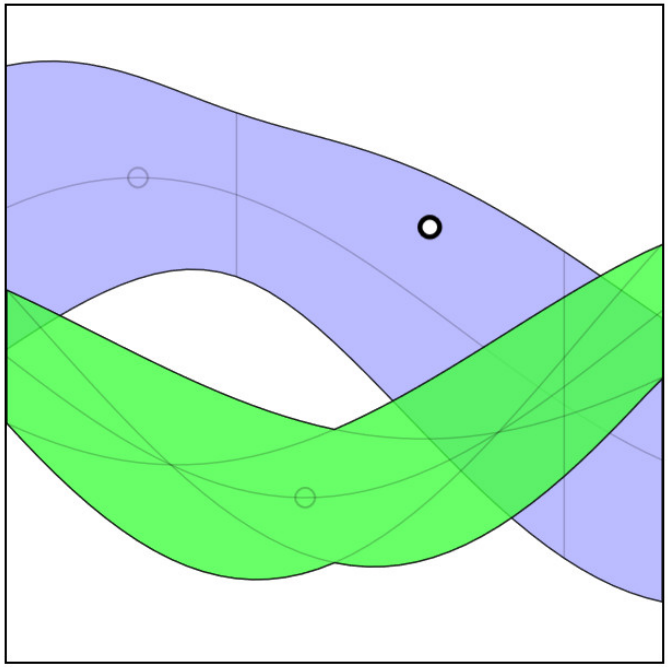
# Our second approach – Demo

Add ellipse Add rectangle Delete object Clear scene

Set color >



Original image



Radon transform

Available online at `https://locsi.web.elte.hu/radon/`

# Our second approach – Verdict

| | Client, Anal |
|---|---|
| detailed transform | ✗ |
| easily accessible | ✓ |
| interactive editing | ✓ |
| real-time transform | ✓ |

# Table of Contents

# Further Research

- Webpage development (descriptions, links)
- More objects (e.g. triangle), more colors
- “dark mode”

- More detailed transform (e.g. level lines)

# Summary

| | Matlab... | Ser, Num | Cli, Anal |
|---|---|---|---|
| detailed transform | ✓ | ✓ | ✗ |
| easily accessible | ✗ | ✗ | ✓ |
| interactive editing | ✗ | ✓ | ✓ |
| real-time transform | ✗ | ✗ | ✓ |

# The End

**Cím:** A Radon-transzformáció valós idejű webes vizualizációja

**Szerző:** Lócsi Levente
ELTE IK Numerikus Analízis Tanszék

**Alkalom:** Bolyai Kollégium, Informatika Szeminárium
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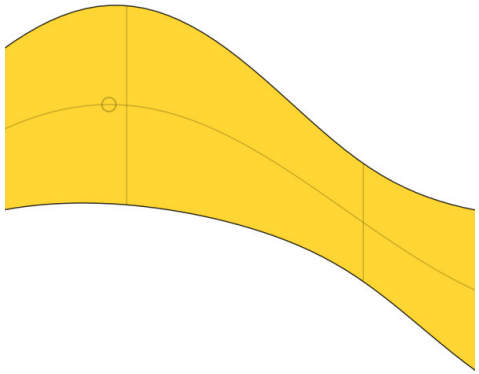

# THANK YOU
# FOR YOUR ATTENTION!

EFOP-3.6.3-VEKOP-16-2017-00001: Talent Management in Autonomous Vehicle Control Technologies – The Project is supported by the Hungarian Government and co-financed by the European Social Fund.

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